Exam Kaleidoscope Mathematics
Part 3: Probability and Statistics

5 Nov. 2019

Clearly write your name and student number at the top of the paper! This exam includes 4 questions. The total number of points you can earn is 90 and you get 10 points for free. Show your workings and explanations for each question, simply stating an answer is not enough. Show why your answer is correct!

Question 1 (25 points)
Carole has an urn containing 5 balls numbered $1, 2, 3, 4$ and 5. She randomly selects a ball, we call the number on the ball that she drew S.

a) (15 pts): What values can S take? Find the value of the cumulative distribution function $F_S(s)$ of S for $s = 1$ and $s = 5$.

b) (10 pts): Let E_2 be the event that S is at least 2. What is E_2^c? Determine $\mathbb{P}(E_2)$ and $\mathbb{P}(E_2^c)$.

Question 2 (20 points)
Alice claims to have a fair coin. She flips the coin 3 times and tells you what the total number of heads was, we call this random variable X. Let the null–hypothesis H_0 be that the coin is fair, we want to see if we can reject H_0 based on the data. Can any outcome of X be used to reject H_0 given a significance level $\alpha = 0.2$?

Question 3 (25 points)
Bob has three different dice: One die with all the even numbers replaced by the number 1, one die with all the even numbers replaced by the number 3, and one die with all the even numbers replaced by the number 5. He randomly selects one of his three dice and rolls it, let the outcome of this roll be the random variable X. For $i = 1, 3, 5$, let E_i be the event that he selected the die with all even numbers replaced by i.

a) (10 pts): Calculate $\mathbb{P}(X = 3|E_3)$.

b) (15 pts): Calculate $\mathbb{P}(E_3|X = 1)$.

(Question 4 on next page)

Question 4 (20 points)

Consider a Markov chain defined on two states labelled 1 and 2 with transition matrix

$$P = \begin{bmatrix} p_{1\to 1} & p_{1\to 2} \\ p_{2\to 1} & p_{2\to 2} \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}.$$

a) (10 pts): Is this a valid transition matrix? Why or why not?

b) (10 pts): Find a stationary distribution for the Markov chain. (Hint: this means you have to find a row vector $\pi = \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix}$ such that $\pi P = \pi$.)